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## Introduction

TODO: Dave can do the ISJ version.

# 1 Conservation laws

In the physical sciences conservation laws provide a basic framework within which theories are cast. They are so fundamental that it is hard to conceive of any mathematically expresseed system of mechanics that does not rest on such laws.

Do similar laws exist in economic theory?

In a sense yes, where they are rather disparagingly termed 'accounting identities' and regarded as something pretty trivial. I want to argue that this view of them as something trivial is misplaced, and that they can actually tell us a lot more about the nature of social relations and their degree of constrainedness than is generally realised.

Marx, to an extent greater than is generally recognised, wanted to establish a theory of the capitalist economy informed by the laws of physics. This comes accross in several way: his avowed aim to write a book on the 'laws of motion' of capitalism; his distinction between the concept of labour and labour power; his presentation of value as the crystalisation of human energy; and his analysis of commodity exchange as an equivalence relation.

Marx said that in Capital he was investigating the 'laws of motion' of capitalism. This might be understood as only a metaphor derived from physics, but we think that it is worth taking it seriously. If you think of the time he was writing - the 1860s, one of the key recent discoveries of physics was the idea of the conservation of energy. The conservation of energy had been formalised by Helmholtz and Grove in the late 1840s. This held that although energy might appear in various forms : heat, motion, gravitational potential, it was conserved in its exchange between these forms.

Marx's initial argument in capital, before he derives labour power as the source of surplus value is similar. Value is neither created nor

destroyed in the exchange process, but can only change its form. His argument asserts in effect a law of the conservation of value in exchange.

Think of the distinction between labour and labour power. This is so directly modelled on Watt's distinction between work and power that it is surprising that this similarity is not more generally recognised. His analysis of commodity exchange is also structured like an analysis of a conservation law[11]. He introduces as an example

20 yards of linen = 1 coat or 20 yards of linen are worth 1 coat,

in this notation he says that the coat plays the role of the equivalent and that it implies also the converse relation

1 coat = 20 yards of linen or 1 coat is worth 20 yards of linen

He then presents what he calls the expanded form of the relation

20 yards of linen = 1 coat

20 yards of linen = 10 lbs of tea, etc.

And goes on from this to state that 1 coat will be equal to 10 lbs tea. What he is doing here is setting out what in modern mathematical terminology is an equivalence relation. For some relational operator $\doteqdot$we say that $\doteqdot$ is an equivalence relation if the relations is commutative, transitive and reflexive, that is

if $a \doteqdot a$

and if $a \doteqdot b$ implies $b \doteqdot a$

and if $a \doteqdot b$ and $b \doteqdot c$ implies $a \doteqdot c$

then the relation $\doteqdot$is an equivalence relation. In his case $\doteqdot$would stand for exchange of commodities. Now equivalence relations are interesting because systems goverened by conservation laws display them. Thus in a many body gravitational problem with a predefined collection of particle masses, the set of possible configurations of particle position and velocities is partitioned into equivalence sets with respect to energy. Within each set all configurations share the same total energy and the conservation of energy prevents transitions of configurations between these sets.

This is in essence what we understand by a conservation law.

We infer the existence of energy as a conserved quantity by the fact that, in closed systems, we never observe transitions between configurations with different total energies.

Marx's demonstration that commodity exchange is an equivalence relation is then used to infer that there is a conserved quantity 'value'

the sum of which is unchanged under the operation of exchanges. Commodity exchange is governed by a conservation law. To borrow von Neumann's much later terminology[**?**], exchange is shown to be a zero sum game.

This may seem a trivial observation, but it leads to an important deduction : that in a conservative system, any surplus of value - profit - must arise from outside of the system and thus that profit must originate from production rather than exchange. Marx argued that the conserved quantity in commodity exchanges is human energy expended as labour and that this labour provides the external input that allows a surplus. At the economy's two ends, production and consumption, the process is non conservative but in between, exchange and the market are a conservative system.

## 1.1 Finance and conservation laws

Although economists are sometimes dismissive of 'accounting identities', these identities can still throw useful light on the financial crisis that has been unfolding these last few years. But to do this we need to uncover the specific laws of motion, that is to say both the conservation laws and the particle dynamics that govern the financial system.

That we have to think of the financial system using tools derived from statistical should be obvious by now. It is over a quarter of a century since it was shown that the regulation of prices by labour values arises directly from statistical mechanical considerations[6].

If we look at the more recent work of Yakovenko on the statistical mechanics of money[4, 5], we can see a similar structure of argumentation - pushed somewhat further in his case by the use of more sophisticated physics. Yakovenko argues that if we treat trades between commodity owners as a random process that conserves money, then you can use statistical mechanics to make deductions about the distribution of money. Money as a conserved quantity randomly transfered in exchanges between agents, models the transfer of energy between gas molecules and it follows that the maximum entropy distribution of money will be a Gibbs-Boltzmann one. Yakovenko shows that this distribution fits the observed distribution of money for most of the population, but that a small minority of very rich people fall off this distribution – their wealth follows a power-law. The log/log PDF of the

Gibbs distribution falls of sharply at high levels of wealth, whereas the power law distribution, a downward sloping straight line extends much further.

The probability of anyone being as rich as Bill Gates or Warren Buffet as a result of simple commodity trading is vanishingly small, so Yakovenko concludes that there has to be some mechanism outside of equivalent exchange that gives rise to their extraordinary wealth: the effect of compound interest which is a non-random process.

Wright has shown that if you partition the population into buyers and sellers of labour power and run agent based simulations, not only does Marx's law of labour value appear as an emergent phenomenon[18], but you also reproduce the combination of Gibbs and power-law distributions of wealth that Yakovenko observes[19].

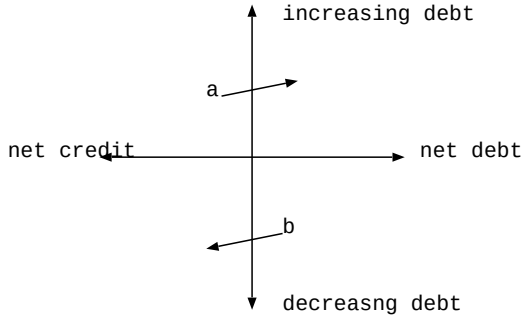
A basic tool of conceptual analysis in statistical mechanics is the concept of phase space. If you consider a collection of particles ( for example stars) in a closed volume, each particle can be described by 6 numbers which specify its position and momentum in terms of a 3 dimensional Cartesian coordinate system : 3 numbers to specify the position and 3 numbers to specify its momentum. We say that each particle has 6 *degrees of freedom*.

So if we have a million particles, for instance if one is considering the dynamics of a galaxy, the system has 6 million degrees of freedom and we can consider these to be a coordinate system such that every possible configuration of molecular positions and moments constitutes a point in this 6 million dimensional space. We call such a space a *phase space*. The laws of motion then specify a trajectory of the whole system through this phase space. The overall system is governed by conservation laws. The mass is conserved, and relative to the center of gravity of the system as a whole the sum of the momenta in each of the directions of our cartesian system must sum to zero.

Why is this relevant to finance?

Well we are again dealing with a system with very large numbers of agents and we have anlogues of position and momentum. The total debt/credit position of an agent is analogous to its mass, and the rate of change of its debt/credit position is analogous to its momentum. Thus if two billion people in the world are enmeshed in debt/credit relations then the whole system can be thought of as a phase space of 2 billion dimensions.

It is impossible to visualise a space with high numbers of dimensions, so there are graphical techniques that people use to reason about them. One trick is to project the high dimensional phase space down onto only two dimensions : for example, position and momentum in in the $x$ direction. Applied to the financial system the phase diagram is as shown here:




Agent $a$ is currently a net creditor, but it is borrowing and as such moving to the right towards becomming a net debtor. Agent $b$ has debts which it is paying off and moving to the left to become a net creditor.

We could show every agent ( firm, individual, state ) as a point on this plane. Or in a more abstract way we could say that there is a probability function $P_{x,y}$ which gives the probability of finding an agent in a given position in the phase plane. Using this probablility function we can formulate two conservation laws analogous to the conservation of mass and momentum.

1. The total 'mass' on the left of the origin equals the mass on the right of the origin. Since the mass to the left is credit, and that to the right is debt, this is another way of saying that the total debt and total credit must balance.

$$0 = \int_{-\infty}^{\infty} x P_x dx \tag{1}$$

2. The total 'mass' above the origin must equal the mass below the origin, that is to say that any growth in debt must be compensated for by a growth in credit. This is an 'equal and opposite reaction', effect.

$$0 = \int_{-\infty}^{\infty} y P_y dy \tag{2}$$

These very basic points establish that there is no net value or net wealth embodied in the financial system and that there is no flow of value into or out of the fiancial system. It shows the fallacy of the conception, popular among some Marxist economists that capital has 'moved into finance' because of the low rate of profit pertaining in industry. This is a fundamental misconception, capital is value, and it can not flow into the financial system, since the sum of value here is always zero. A moments thought about the materiality of value confirms this. Value, labour embodied in physical products, can not be converted into fianancial instruments which are just information structures.

It also shows the fallacy of the idea of the 'money supply' used in orthodox economics. The notion of a supply originates with physical flows like the supply of water to a town, or the supply of cars provided by all the car factories in the world. There are flows in the financial system - flows of agents towards greater debt, but these are exactly balanced by flows towards greater credit positions, so the net flow is zero.

We believe it is better to try and understand the system in terms of its laws of motion. These laws are both the conservation laws 1 and 2 , and the forces acting on the individual particles ( capitals, states etc). We will argue that the entropy of a financial system tends to increase over time. We would expect this of any chaotic system governed by conservation laws, but an examination of the force fields acting on the particles will show why this takes place.

We will define the entropy using the standard Boltzman $H$ formula

$$H = -\int P_x \ln P_x dx \tag{3}$$

and we assert that for a financial system

$$\frac{dH}{dt} \geq 0 \tag{4}$$

Why does the entropy tend to increase?

First a common-sense explanation. Consider a collection of firms or enterprises, distributed on a slightly different type of phase plane diagram, one in which the $x$ axis indicates the gearing ratio of the firm

and the $y$ axis the rate of chang of the gearing ratio[1]. Each firm can be considered a particle subject to forces which determine its rate of change of gearing. There are three cases to consider:

1. A firm whose gearing ratio is rising because its profits are two low even to meet its interest payments to the banks. Such a firm has to borrow more from the bank to stay in operation. This firm is an involuntary borrower.

2. A firm may be voluntarily borrowing because it has a high rate of profit, well in excess of the interest rate, and thus can increase its profit of enterprise by taking on bank loans to expand its business.

3. A firm may reckon that the rate of profit it can earn is lower than the rate of interest, but its current gearing ratio may be quite low so that it is in a position to pay off its debts to the bank with some of its retained profit. A firm may even have no net borrowing and find that it is more profitable to earn interest on its cash than it is to invest it productively. Such a firm is a voluntary lender.

Because the total amount of lending must equal to the total amount of borrowing, if quantity of capital in condition 3 exceeds that in condition 2, the lack of demand caused by the firms in condition 3 failing to invest will automatically create sufficient firms in condition 1 to ensure that the equation 2 is met. The net effect of this is to polarise capital in the phase plane, giving rise to a tadpole shape with a head of productive firms and a tail of rentier firms, who earn their revenue from financial rather than productive assets.

Because a more spread out distribution of a probability density function has a higher entropy, represents more disorder, the net effect of these processes is to enforce equation 4.

In an earlier paper [3] we presented a numerical simulation model based on the methodology developed by Ian Wright that models a simple capitalist economy with a large number of capitalists and workers. The state of indebtedness/credit of each of the firms is tracked as the economy evolves. The capitalists interact via a very simple financial system represented by a single bank which maintains debit and credit

[1] The gearing ratio of a firm is the ratio of its net debt to its total capital. Note that a firm with a net credit position has a negative gearing ratio.

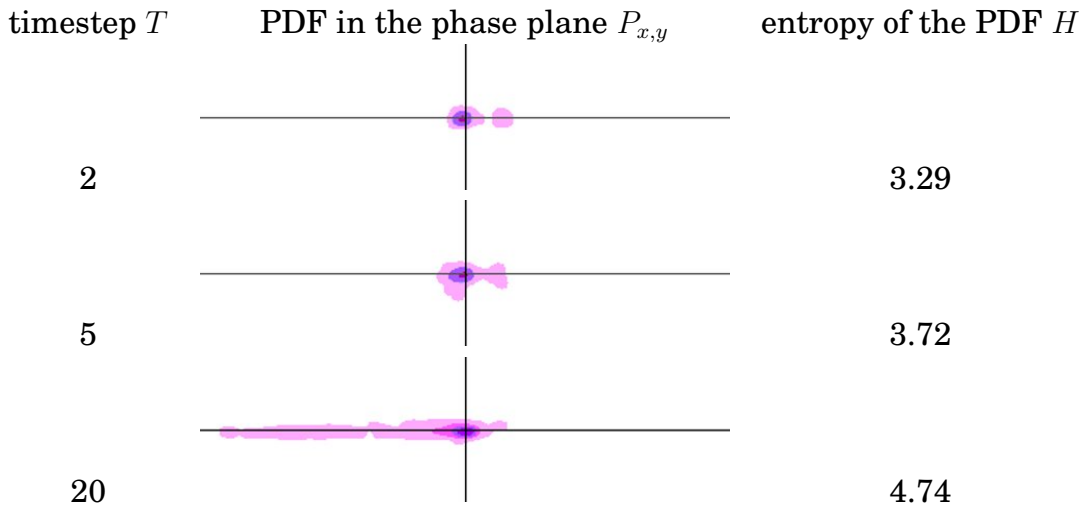



Fig. 1: The formation of a rentier tail due to the polarisation of capitals in the phase plane. The images show a plot of the probability density function of capitals in the phase plane of Gearing Ratio against rate of change of Gearing Ratio with respect to time. The x axis shows the gearing ratio, with positive gearing to the right. The y axis shows the rate of change of the gearing ratio, with increasing gearing above the x axis and decreasing gearing below the axis. As time passes the entropy of the system increases. Note that the colour is non linearly related to the actual value of $P_{xy}$in order to display the areas of very low but non-zero probability in the tail.

accounts for agents based on a certain amount of base money which acts as its reserve. Initially the capitalists are clustered around the origin of the phase plane diagram, but as time passes the distribution becomes elongated with head of relatively indebted firms and long 'rentier tail' of capitalists with a negative gearing ratio. This process is illustrated in figure 1. The increase in entropy over time, calculated using a per pixel binning of the PDF is shown.

The greater is the spread of the rate of profit[2] and the smaller the gap between the rate of profit and the rate of interest the stronger will be this polarisation process and the more rapid will be the growth in financial entropy[3].

The polarisation process generates a tadpole shape in the phase plane with a body of active firms clustered round the origin and a rentier tail of firms that progressively enlongates as the simulation goes on. Why do we get this asymetrical distribution?

Because the gearing ratio of a firm can not for long exceed unity. A firm with more debts than assets is technically bankrupt and will shortly cease trading. There is on the other hand no limit to how negative the gearing ratio of a firm or capitalist can be. That is to say, no limit to how much money a firm can have in the bank.

## 1.2 Partial conclusion.

What the argument above shows is that it is possible to derive a small set of laws of motion that characterise the basic development of a capitalist financial system. The phenomenon that some writers have called financialisation is a inevitable tendancy of these laws of motion. The entropy of the system will increase, polarisation will occur, and a ren-

[2] Technically what we mean here is its coefficient of variation, its standard deviation relative to its mean.

[3] As we said earlier the net lending always sums to zero. As such it is analogous to a motorway with a stream of cars of equal weight heading past one another at 100kph in opposite directions. The sum of the momentums of the cars is zero. But if we square their positive or negative velocities times their weights we get the energy stored in the traffic. One can view the sum of the squares of the lending positions of all agents as a bit like energy in that it does not sum to zero as the activity of the financial system increases but with the important proviso, that this quantity is not conserved. It can grow without immediate limit. We will explain later why this growth in financial energy leads to crises.

tier class will be precipitated out, unless social work is done to curtail the growth of entropy.

Commodity exchange is a conservative system, ie, one governed by conservation laws. This does not apply to taxation. Taxation is a non-equivalent transfer of value. Heavy taxation of the rentier class is a form of social work that reduces the entropy of the sytem.

The introduction of a planned economy is an even more powerful form of social work. In a planned economy the degree of chaos and disorder is reduced and coherent patterns are established which curb the growth of entropy characteristic of the free market.

## 1.3 Relation to the productive economy

The model above is very simple, it involves no inter-bank lending nor the issue of any complex financial instruments. All growth in debt arises from the behavior of basic actors in the real economy : capitalists and workers. But it is still able to generate the polarisation of the capitalist class into productive capitalists and financiers.

In a physical system, as it moves from a low entropy state to a high entropy state we are in principle able extract work from it. Is there any equivalent in the financial system?

Does it release any analogue of free energy as it evolves?

It is a commonplace observation that an economy with a low initial level of debt can grow rapidly as the debt builds up, but what is happening here?

We have said that there is no value absorbed in or contained in the financial system. The financial system is an information structure recording the mutual obligations of agents. But this does not mean that there may not be real value correlates of financial relations. If we look at all the firms with positive gearing - the integral over the right hand side of the phase plane - then these firms have all absorbed real resources from the left hand side of the plane. When capitalists on the bottom half of the phase plane fail to invest and capitalists on the top half do invest in excess of their income, the financial system sets up a system of obligations between the debtors and the saving capitalists. But at the same time, the firms on the bottom are provided with a market for their output on the top of the phase plane. There a transfer of real commodities from the bottom to the top.

A firm on the bottom, produces 10,000 buses containing perhaps 100 million hours of labour which are bought by agents on the top half in return the bus company obtains not value but a credit account at the bank. This is not strictly speaking a commodity exchange. A credit account in a bank is not value. Had they been paid in gold and got back 1 million oz of gold bullion, that would have been a commodity exchange, an exchange of equivalents, since the gold would have contained real value in the form of the labour required to make it.

The sale of the buses for say €1,000 million is a non equivalent transaction in real terms since embodied labour is transfered between owners without a compensating movement of labour in the other direction. And, by our previous assumption, it is a transfer between a bus firm who were deciding to save $1,000$ million rather than invest it productively. This transfer is thus one which would not have occured in the absence of the credit system. Had gold coins been the medium of exchange, the million oz of gold would have lain in their safe at the bus factory and those purchasers who lacked cash ( those who are on the right of the phase plane diagram ) would not have been able to buy the buses. The sale would thus never have taken place, and the total output of buses would have had to be 10,000 lower. Buses could still have been sold to firms who had ready cash, but this would have been a smaller number.

Those of you who have read Marx's Capital[11] will recall that he analysed the circulation of commodities as being of the form

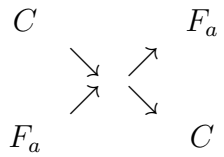
$$C \rightarrow M \rightarrow C$$

, commodity , money, commodity. He argues that this form already contains the possibility of crisis[4]. The potential crisis is caused by the formation of gold hoards, gold which is withdrawn from circulation and

[4] Nothing can be more childish than the dogma, that because every sale is a purchase, and every purchase a sale, therefore the circulation of commodities necessarily implies an equilibrium of sales and purchases. If this means that the number of actual sales is equal to the number of purchases, it is mere tautology. But its real purport is to prove that every seller brings his buyer to market with him. Nothing of the kind. The sale and the purchase constitute one identical act, an exchange between a commodity-owner and an owner of money, between two persons as opposed to each other as the two poles of a magnet. They form two distinct acts, of polar and opposite characters, when performed by one single person. Hence the identity of sale and purchase implies that the commodity is useless, if, on being thrown into the al-

saved. This interruption of the circuit $C \to M \to C$ by money lying idle means that goods can not be sold: 'No one can sell unless some one else purchases'.

With credit the circuit $C \to M \to C$ is replaced from the standpoint of the seller with one of the form $C \to F_a \to C$ where $F_a$ is a financial asset : a bill of exchange, or a record of credit with a bank. If both the purchaser and the seller have financial assets this is the same as the old monetary circulation:




The financial asset has just changed places, but if we have purchase

chemistical retort of circulation, it does not come out again in the shape of money; if, in other words, it cannot be sold by its owner, and therefore be bought by the owner of the money. That identity further implies that the exchange, if it do take place, constitutes a period of rest, an interval, long or short, in the life of the commodity. Since the first metamorphosis of a commodity is at once a sale and a purchase, it is also an independent process in itself. The purchaser has the commodity, the seller has the money, i.e., a commodity ready to go into circulation at any time. No one can sell unless some one else purchases. But no one is forthwith bound to purchase, because he has just sold. Circulation bursts through all restrictions as to time, place, and individuals, imposed by direct barter, and this it effects by splitting up, into the antithesis of a sale and a purchase, the direct identity that in barter does exist between the alienation of one's own and the acquisition of some other man's product. To say that these two independent and antithetical acts have an intrinsic unity, are essentially one, is the same as to say that this intrinsic oneness expresses itself in an external antithesis. If the interval in time between the two complementary phases of the complete metamorphosis of a commodity become too great, if the split between the sale and the purchase become too pronounced, the intimate connexion between them, their oneness, asserts itself by producing — a crisis. The antithesis, use-value and value; the contradictions that private labour is bound to manifest itself as direct social labour, that a particularised concrete kind of labour has to pass for abstract human labour; the contradiction between the personification of objects and the representation of persons by things; all these antitheses and contradictions, which are immanent in commodities, assert themselves, and develop their modes of motion, in the antithetical phases of the metamorphosis of a commodity. These modes therefore imply the possibility, and no more than the possibility, of crises. The conversion of this mere possibility into a reality is the result of a long series of relations, that, from our present standpoint of simple circulation, have as yet no existence. [11]Chapter 2, Section 2.

on credit we get:

$$\begin{array}{ccccc} C & & & & F_a \\ & \searrow & \nearrow & & \\ & \nearrow & \searrow & & \\ a^\dagger & \rightarrow & \rightarrow & C + F_l & \end{array}$$

Here the purchase is funded by the spontaneous creation of a financial asset/liability pair by the debt creation operation $a^\dagger$ giving rise to $F_a$ and $F_l$. Within this whole process real value is conserved, since the only real value entering was $C$ and $C$ leaves having merely changed owner whilst the liability and asset cancel: $F_l + F_a = 0$.

The 'work' done by credit is thus to allow the production and distribution of goods embodying real labour that could not otherwise occur given the private organisation of production. Commodity exchange requires and exchange of equivalents. Where these equivalents are not in the hands of the purchasers, the financial system allows transfers that are in real terms non-equivalent exchanges, whilst creating instead a symbolic recompensation for the seller.

This symbolic recompensation in Euro or Dollars is a theoretical command over future labour. The borrower in this case is the *real appropriator* of the labour value embodied in the capital goods they acquire. The lender obtains merely a *formal or symbolic appropriation* of value.

Private ownership creates a potential barrier to the movement of goods which credit overcomes. The increasing entropy of the financial system reflects the 'work' that has been extracted in overcoming this barrier and is why it appears to 'create wealth'. It allows the creation of wealth by labour whose expenditure would otherwise have been inhibited by the private organisation of the economy. If one were to ask a banker what productive role they paid in the economy the answer would probably be in terms of the banks providing the finance that the economy needs[5]. Money according to Smith is the ability to com-

[5] Now let's turn to the purpose of banks in a capitalist economy. Finance is an intermediary good: You cannot eat it, experience it, or physically use it. The purpose of finance is to support other activities in the economy. Banks are meant to allocate capital (funds) to the best possible use. In a capitalist economy, this means allocating money to the people or entities that will create the greatest wealth for the overall

mand the labour of others[6]. The provision of credit gives a capitalist the authority or permission to commandeer part of the pool of social labour to his project. In this sense the provision of a line of credit by a bank is like any other official act of giving permission, like for example a building control office issuing a permit allowing a house to be built. But the right to hand out such permissions does not make the person handing them out productive. It is bricklayer's and carpenters that actually produce the house, not the bureaucrat who signs it off. When such permissions are in demand, the official handing them out may ask for a cut: 'You want to have this house built, you know your application might go much more smoothly were you to show your generosity in some way'. In an analogous fashion bankers ask for their cut: interest. At one time the charging of interest ( usury ) as regarded as the moral equivalent of an official taking a bribe. With the rise of bankers to political dominance, their very wealth, obtained in this way comes to be seen as a token of social respectability[7].

### Contradiction between real and formal appropriation of value

Why do capitalists seek profits?

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society. At the same time, risk management is supposedly a primary skill for bankers. When capital is allocated well and available to wealth creating entities, societies flourish. When capital is poorly allocated, economies can collapse.[9]

[6] Wealth, as Mr Hobbes says, is power. But the person who either acquires, or succeeds to a great fortune, does not necessarily acquire or succeed to any political power, either civil or military. His fortune may, perhaps, afford him the means of acquiring both; but the mere possession of that fortune does not necessarily convey to him either. The power which that possession immediately and directly conveys to him, is the power of purchasing a certain command over all the labour, or over all the produce of labour which is then in the market. His fortune is greater or less, precisely in proportion to the extent of this power, or to the quantity either of other men's labour, or, what is the same thing, of the produce of other men's labour, which it enables him to purchase or command.([14] Chapter 5)

[7] This disposition to admire, and almost to worship, the rich and the powerful, and to despise, or, at least, to neglect persons of poor and mean condition, though necessary both to establish and to maintain the distinction of ranks and the order of society, is, at the same time, the great and most universal cause of the corruption of our moral sentiments. That wealth and greatness are often regarded with the respect and admiration which are due only to wisdom and virtue; and that the contempt, of which vice and folly are the only proper objects, is often most unjustly bestowed upon poverty and weakness, has been the complaint of moralists in all ages. ([13]page 53)

On the one hand it is something that is imposed by the nature of competition. Only profitable firms survive, the more profitable they are the more likely they are to survive, so a mechanism analogous to natural selection establishes the profit motive as the driving force of the system. But there is another way of looking at this. Whatever its historical form, gold, banknotes, bank accounts, money has been the power of command over labour. In all class societies, members of the upper class has been driven to increase their power over the labouring classes. Slave owners tried to obtain as many slaves as they could. Feudal landowners sought to do this indirectly by building up their landed estates, since attached to the land were serfs. Capitalists do it by accumulating money. A billion euros gives a capitalist command over about 100,000 person years of labour, say 2000 working lifetimes of indirect labour in commodities, perhaps twice that if they employ people directly. The more money you have, the more people are at your command.

Building up a bank balance gives you a symbolic command over lots of future labour but this is different from really having the product of this labour, as the fable of Midas long ago pointed out. But because of the nature of credit, the thrifty symbolic apropriator depends for his very existence on the spendthrift debtors.

The credit system socialises this dependence, makes it impersonal, so that money seems to simply represent abstract value, abstract command over labour. But the labour it is command over, is that of the debtor classes as a whole. The sub-prime crisis, like all credit crises brings to the fore the fragile and mirage like quality of this symbolic command, which can grow beyond any possibility of conversion into a real appropriation.

Think back to the phase diagram. The symbolic wealth of the rentier classes is the tadpoles tail. As it wiggles and extends, it drives the head towards the right towards the wall of bankruptcy( Fig 2). Hit the wall and the credit creation operator $a^\dagger$ becomes its terrible twin the anihilation operator $a$ which cancels out the debt of the bankrupt at the same time as it wipes out the asset of the creditor.

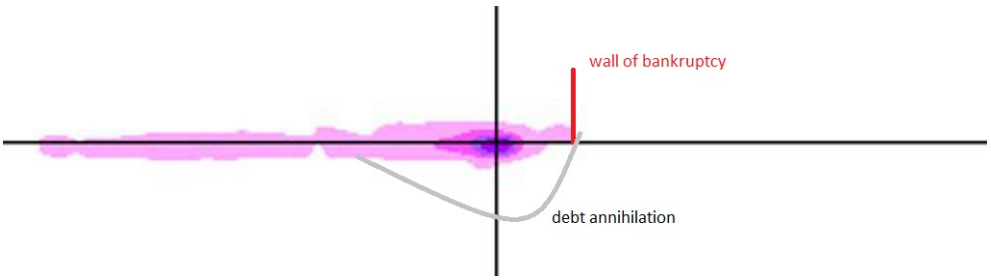


Fig. 2: Capitals pushed against the wall of bankruptcy lead to debt/credit annihilation.

## 2 Contradiction between value conservation and the signature of capital.

The most obvious characteristic of capital—one might call it the signature of capital—is its ability to undergo exponential growth. Marx characterized this signature using the notation $M - C - M'$, where $\Delta M = M' - M > 0$, and attempted to explain how such expansion is possible.

It could not, he argued, come about in the field of commodity exchange, since this was governed by a law of conservation of values. Thus, he argued, profit could only be explained outside of the realm of commodity exchange, by the exploitation of labour in the production process. In the capitalist factory—where Freedom, Equality and Bentham do not prevail—the working day is extended beyond the time required to produce the workers' means of subsistence, in order to provide a surplus that funds profits.

Ian Wright [18, 19] has shown that this model can be validated in computer simulations, with some very simple assumptions.

But Marx's analysis only partly answers the problem of "where the money comes from". He explained how capitalists obtained a net income from their capital, but this was only half the problem. If the capitalists follow the maxim ascribed to them by Marx—"Accumulate, accumulate! That is Moses and the prophets!"—then the signature of capital $M - C - M'$ extends into

$$M - C - M' - C' - M'' - C'' - M''' \ldots$$

which requires exponential growth in the quantity of money. In the

19th century, the British economy, like most others, depended on precious metal for its monetary base. An exponential growth in the quantity of money implies the same sort of growth for gold stock. But if we look at historical data for the growth of the world gold stock, we find that during the 19th century it was growing at well under 1% per annum. Given that the British economy grew at over 2% a year, there was a discrepancy between the growth of gold and the growth of commodity circulation.

Since gold stocks could not grow fast enough to support the expansion of the economy, capitalists had to resort to commercial bills. An Iron Master taking delivery of coal would typically write a bill of exchange, a private certificate of debt, promising to pay within 30 or 90 days.

Payment of wages would generally have to be done in cash. Capitalists have tried at times to pay wages in tokens redeemable only at company stores ("scrip") but legislation by the state, eager to maintain its monopoly of coinage if not to defend the interests if the workers, tended to put a stop to this. Payment in cash represents a transfer from the safes of capitalists to the pockets of their employees, with a corresponding cancellation of wage debts. At the end of the week, the wage debt has been cleared to zero, and there has been an equal and compensating movement of cash.

Workers then spend their wages on consumer goods. For the sake of simplicity we assume that there is no net saving by workers so that in the course of the week all of the money they have been paid is spent. This implies that immediately after pay-day, the money holdings of the workers are equal to one week's wages. If these wages were paid in coin this would have set a lower limit to the quantity of coin required for the economy of function.

When workers spend their wages on consumer goods they transfer money only to those firms who sell consumer goods—shopkeepers, innkeepers and so on. We can expect these firms not only to make up the money they had paid out in wages, but to retain a considerable surplus. The final sellers of consumer goods will thus end up with more money than they paid out in wages. From this extra cash they can afford to redeem the bills of exchange that they issued to their suppliers.

In the absence of bank credit, suppliers of manufactured consumer goods would be entirely dependent for cash on money arriving when the

bills of exchange, in which they had initially been paid, were eventually redeemed by shopkeepers and merchants. The payment situation facing raw materials firms was even more indirect: they could not be paid unless the manufacturers had sufficient cash to redeem bills of exchange issued for yarn, coal, grain, etc.

| Period | Stock (million troy oz.) | Annual growth (percent) |
|---|---|---|
| [4pt] 1840–1850 | 617.9 | 0.27 |
| 1851–1875 | 771.9 | 0.89 |
| 1876–1900 | 953.9 | 0.85 |
| 1901–1925 | 1430.9 | 1.64 |
| 1926–1950 | 2130.9 | 1.61 |
| 1951–1975 | 3115.9 | 1.53 |
| 1976–2000 | 4569.9 | 1.54 |

Tab. 1: Growth of the world gold stock, 1840 to 2000

The process of trade between capitalists leads to the build-up of inter-firm debt. We suggest that the total volume of inter-firm debt that could be stably supported would have been some multiple of the coinage available, after allowing for that required to pay wages. If one takes the aggregate of all firms the ideal signature of this process can be represented as:

$$M \rightarrow [C \Rightarrow (C + \Delta C)] \rightarrow M + \Delta M$$

where $[C \Rightarrow (C+\Delta C)]$ represents the production process that generates a physical surplus of commodities after the consumption needs of the present working population has been met. If there is no new issue of coin by the state then the $\Delta M$ cannot be "real money"; rather, it must be in the form of bills of exchange and other inter-firm credit.

For the capitalist class considered as a whole this should not be a problem since the $\Delta M$ is secured against the accumulated commodity surplus $\Delta C$. There is a net accumulation of value as commodities, and accounting practice allows both the debts owed to a firm and stocks of commodities on hand to be included in the value of its notional capital. As the process of accumulation proceeds in this way the ratio of commercial debt to real money will rise. If the period for which commercial

credit is extended remains fixed–say at 90 days—then a growing number of debts will be falling due each day. If these have to be paid off in money, then a growing number of firms will have difficulty meeting their debts in cash.

The basic contradiction between capital's exponentially growing need for money and the much slower growth of gold production led to a series of transformations of the monetary system during the 19th and 20th centuries[8].

1. Gold was supplemented by commercial credit. This displaced gold from transactions between capitalists.

2. Commercial credit was supplemented by the discounting of bills of exchange by the banks.

3. Payment in bills of exchange was largely replaced by payments by cheque and commercial credit by bank credit.

4. Gold coins were withdrawn from circulation to be replaced by banknotes with gold only used in settlements between international banks. This meant that wage payments no longer depended on precious metal.

5. National currencies were then completely removed from the gold standard, and state notes became the base money. This was completed by the withdrawal of the dollar from the gold standard in the 1970s.

6. With the development of computerisation in the 3rd quarter of the 20th century it became practical to pay wages directly into bank

[8] One of the principal costs of circulation is money itself, being value in itself. It is economised through credit in three ways.

A. By dropping away entirely in a great many transactions.

B. By the accelerated circulation of the circulating medium. ... On the one hand, the acceleration is technical; i.e., with the same magnitude and number of actual turnovers of commodities for consumption, a smaller quantity of money or money tokens performs the same service. This is bound up with the technique of banking. On the other hand, credit accelerates the velocity of the metamorphoses of commodities and thereby the velocity of money circulation.

C. Substitution of paper for gold money.

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accounts. This meant that state base money circulating could be substantially less than the monthly or weekly wage bill.

7. Finally with the general issue of credit cards, the credit system spread from the capitalist class to all classes in society.

When money was still gold, this gold was value - it was embodied labour and had a value internationally because in all countries the production of gold required a great deal of labour. With the modern system of national and supernational currencies money is no longer value. No significant work goes into the printing of €100 notes. Why then can they function the same way that gold used to?

Central bank notes used to be issued under the gold standard, but these were just tokens for gold, and could be redemed for bullion on demand at the central bank. There is no promise by the ECB to redeem Euros to gold at any fixed exchange rate[9]. Since there is no definite link between the Euro and gold or between the Pound and gold, how can these currencies function as a measure of value and medium of exchange?

Why are they worth anything?

Well for one thing they are legal tender, but what does this mean and why is it important?

Throughout the Euro zone the following is held to apply in cases where a payment obligation exists:

- Mandatory acceptance of euro cash,; a means of payment with legal tender status cannot be refused by the creditor of a payment obligation, unless the parties have agreed on other means of payment
- Acceptance at full face value; the monetary value of a means of payment with legal tender status is equal to the amount indicated on the means of payment
- Power to discharge from payment obligations; a debtor can discharge himself from a payment obligation by trans-

[9] though collectively the central banks of the Euro area held over €430 billion in gold reserves. These can potentially be used in settlement with other central banks to settle foreign trade debts, but in practice there is little or no intervention by the ECB using gold to support the value of the Euro.

> ferring a means of payment with legal tender status to the creditor.[1]

The circulation of the Euro is legally enforced in the relationship between shops and customers, but between businesses they can in principle both agree to settle obligations in something other than Euros. But with retail commerce and all taxes payable in Euros, its circulation is effectively enforced.

But there is a difference between the previous generation of state monies like the Franc or D-mark and the Euro. The previous generation fell into the general category of state token monies. This is a very old category of money[8, 10, 17]. Marx's analysis of money focused on precious metal money, which was prevalent in early modern Europe. But in China the monetary system was from a much earlier stage based either on copper tokens or on paper notes[15, 16] and it is arguable that for much of the Roman Empire the denarius was little more than a copper token with merely a symbolic coating of silver[2]. In such a system the circulation of money goes as follows

| *State* | *Pay* | *Lackeys* | *Purchases* | *Producers* | *Tax* | *State* |
|---|---|---|---|---|---|---|
| **Emperor/** | | | | | | **Emperor/** |
| **King** | $\rightarrow$ | **Soldiers** | $\rightarrow$ | **Artisans** | $\rightarrow$ | **King** |
| $\uparrow$ | | **Bureacrats** | | **Peasants** | | $\downarrow$ |
| $\nwarrow$ | | $\leftarrow$ | | $\leftarrow$ | | $\swarrow$ |

In such a system the empire or state imposes the circulation of its token currency by obliging the producers to pay taxes in money. Since the producers must 'render unto Ceasar', they are forced to sell their product to the employees of Ceasar. The state creates money tokens with which it pays its employees. The state employees willingly work for the state in return for these tokens knowing that these tokens will enable them to command the labour of others in their turn. The state thus breaks down the self sufficient or barter economies of the countryside and enforces the spread of commodity exchange. Forestater gives a dramatic account of how this process was enforced in the British Empire[7].

In a pre monetary tax system, the tax is levied in the form of a direct duty on the population to work for or deliver goods to the state. In

this case the real appropriation of labour by the state is directly visible. In a monetary tax system with token money the *real appropriation* of surplus labour occurs when soldiers and other state employees deliver goods and services to the Emperor. There is a distinct *formal or symbolic appropriation* when the taxes are levied on the producers. This is similar to the relationship that we previously analysed between formal and symbolic appropriation in the credit system, but with this difference: the formal tranfer in the tax system has no pretence of equivalence.

The value of a state token money is based on something historically prior to commodity production, something that goes right back to the earliest state forms : the power of the state to command the labour of its inhabitants. The value of the Swedish Krona is set by the fact that the Swedish state (Crown) directly or indirectly appropriates more than half the labour in the Kingdom[10]. The sum in Krona paid out for that directly social labour has its value set by this labour that the Crown directly commands. This rate of exchange between royal tokens and labour sets the monetary equivalent of labour time in Sweden, which then operates via the medium of commodity trade within the remaining private part of the economy. The direct royal command over the labour of his subjects is then symbolically appropriated by the capitalist class in the form of Krona credits in the Svenska Handelsbanken etc to give them command over the privately employed working class.

For a monetary system like this to work, you need a clearly defined and controlled territory of the empire, an efficient tax system, and a state that commands a substantial portion of the total labour of society. And that state has to have the sovereign right to issue its own currency.

## Laws governing the distribution of profit rates

In the section above we have illustrated the distribution of gearing ratios, $G = D/K$, i.e. the amount of net debt relative to total capital, in the firm sector. Now we consider the distribution of profit rates,

[10] This is a slight simplification, public expenditure is over 53% of GNP, but a part of that is indirect appropriation of labour as social protection, where individual citizens are paid social benefits in Krona.

$R = P/K$, i.e. the flow of profit (before interest and dividend payments) relative to total capital, in the firm sector. TODO: distribution over capital.

This distribution is of importance to the dynamism of a capitalist economy. The rate of profit is an indicator of the rate of return on capital invested in different lines of production and therefore affects the investment decisions of each firm. The mean of the distribution also sets the upper limit to the rate at which the total capital stock can grow, i.e. when the total mass of profit is reinvested.

Figure 3 illustrates the entire invested capital stock as distributed over different rates of profit. We can deduce two constraints on this distribution. First, due to the mechanism of bankruptcies, the fraction of capital making an immediate loss rapidly shrinks along the negative end of the x-axis. This constrains the distribution from below. Second, by the stochastic labour theory of value, the mean of the distribution is well approximated by the ratio of surplus labour to labour required to reproduce the capital stock. The mean can then be shown to be governed by rates of change and the investment level in the production system CITE. More precisely, the mean of the distribution has a dynamic steady-state value,

$$R^\star = \frac{l + p + \delta}{j}, \tag{5}$$

where $l$ and $p$ denote the rate of growth in labour and productivity, respectively, $\delta$ denotes the depreciation rate of the capital stock, and $j$ is the ratio of gross investments to profits. Figure TODO shows how the steady-state rate has governed the average profit rate in the US economy over the period XXXX-2009. In periods when $R^\star$ rises, the distribution in Figure 3 spreads towards the positive direction of the x-axis. In periods when $R^\star$ falls, the economic system is undergoing pressures to reduce the dispersion since the distribution is pushed towards the lower end of x-axis in Figure 3. If a large fraction of the distribution is pushed below the rate of interest, the firm sector is dragged into a crisis of profitability. Firms react by reducing investments; those with high gearing ratios find that profits cannot match interest payments, those with low ratios find it more profitable to earn income from interest.

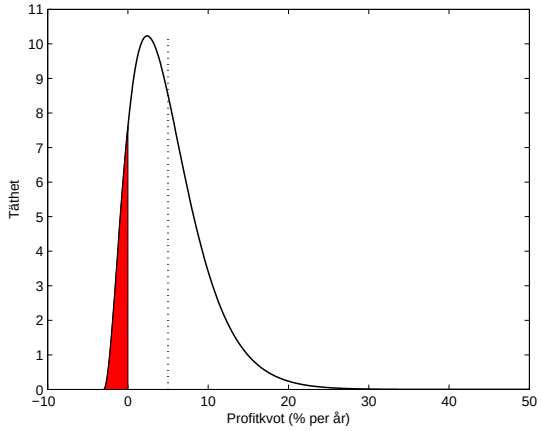


Fig. 3: REDO FIG. Red area indicates fraction of capital making an immediate loss. The vertical dotted line indicates the mean of the distribution.

## Formation of interest

The interaction between the rate of profit and interest outlined above demands further an analysis of the formation of interest in the capitalist banking system. The specific feature of this system is deposit taking.

A deposit-taking banker is in a much more perilous position since he accepts cash over and above his own capital which he then lends out. Because he has lent out cash that was not his own capital, and is under an obligation to encash deposits on demand (or after some fixed warning period), he can easily become insolvent. The remaining cash he holds in his safe is never enough to meet his maximum obligation to his creditors. Should the day dawn on which too many of them demand their money back, he is lost.

Suppose we model this as a stream of customers arriving at the banker's till at random intervals. Each customer either makes a deposit or a withdrawal. The customers may make a withdrawal of any amount up to their current credit balance. We further assume that in a steady state customers are as likely to make a deposit as to make a withdrawal. Then the more customers that a bank has, the smaller

will be the proportional variation in the withdrawals from day to day.

As the number of customers rises, the variation in the amount withdrawn in any week falls, and so too does the maximum withdrawal that can be expected. A very small bank would have to keep all its deposits in the safe as an insurance against having to pay them out, but a bank with 20,000 customers might never see more than a few percent of its cash deposits withdrawn in any week. A bank with that number of customers could safely issue as loans several times as much in paper banknotes as the coin that it held in its vaults, safe in the knowledge that the probability of it ever having to pay out that much in one day was vanishingly small. Thus with a starting capital of 10 million the bank could lend out 200 million after building up its network of customers. This creation of new paper money by the banks was the hidden secret behind the signature of capital.

The cost to a bank of making a loan is related to the likelihood that the reserves left after the loan will be too small to cover fluctuating withdrawals. If this happens the bank may lose its capital.

Consider a random variable, $W$, which is the maximal excursion of reserves from their mean position during a year, due to random deposits and withdrawals by customers. Let us assume that $W$ follows a Gaussian distribution, with a standard deviation of 1,000,000. Figure TODO illustrates this. Suppose that the banker had a capital of 5,000,000 and that he would lose his capital if the bank failed. Then, if he started with reserves of 3 million, making a loan of 1 million would reduce the reserves to 2 million and the loan would have an expected cost of $P_w \times 5,000,000$, where $P_w = \Pr\{-3 \text{ million} < W \leq -2 \text{ million}\}$. This amounts to an expected cost of about 125,000, which sets a lower limit on the interest it would be rational for the banker to charge for the loan, namely 12.5% in this case.

Different banks will charge different rates of interest, but through the pressures of bankruptcies the lower safety rate is likely to emerge from capitalist banking practices. As the ratio of reserves to deposits fall, this pressure would be reflected in higher interest rates. There would thus be an inverse relationship between the reserve to deposit ratio and the interest rate.

TODO: instability of small number of depositors or creditors to the bank, interbank system.

## Long term tendencies

TODO: integrate a) states and b) 'foreign sector' before this?

From the standpoint of exchange being a conservative system, the exponential growth of value implied by compound interest has to be a temporary *disequilibrium* phenomenon - tied to an exponential growth in whatever is the source of value. If the expenditure of human energy is the source, then the accumulation of capital value must depend on a similar exponential growth of the working population.

We know that historically the process of industrialisation has combined rapid accumulations of capital values with an exponential growth in the working population. But we also know that societies undergo a demographic transition once they have developed further - with a shift to zero or negative rates of natural population growth. The end to population growth is bound eventually to make capitalism unviable.

If we look at Western Europe and Japan, the natural rate of population growth is very low. In the face of this constraint we see set of displacement processes:

1. Pressure of immigration from areas of high birth rate. On the one hand this is what one would expect simply as a diffusion process, but there is also political 'pressure' by capitalists interests to reduce barriers to movement of labour in order to allow continued capital growth.

2. As it becomes harder to reinvest profits in new labour employing activities, there is an increasig dislocation between the apparent accumulation of capital in financial instruments and the reality in which the surplus product is actually being socially used by the state.

There is a fundamental conflict between the attempt to maintain a mechanism of symbolic appropriation of the surplus by private owners and the European demographic conditions which mean that little real accumulation is now possible in the private sector. The greater part of the social surplus product now has really to be appropriated by the state as representative of society. The continuation of a private claim on this surplus becomes actually infeasible. The conflict between an exponential growth of financial claims and a stagnant population upon whom those claims rest arguably lies at the heart of the 'Euro Crisis'.

## Greek debt

For example we know that the Greek trade deficit with Germany and the German trade surplus with Greece must sum to zero. It is impossible to reduce one without reducing the other. If austerity in Greece reduces their deficit with Germany, there must be a reduction in German exports. If the objective is to reduce the trade imbalance between Germany and Southern Europe, it would be better to give Germans longer and better paid holidays so that they could spend more on mediteranean holidays rather than impoverishing both sides.

It is also helpul to look at the problem of state debts this way. If at the end of a period the state is to reduce its liabilities and improve its net worth, this necessarily implies a reduction in the assets held by the state's creditors. How to reduce state debt and how to impoverish the holders of state debt are one and the same problem. In this light, a choice of options become visible

- debts can simply be repudiated,
- inflation can be engineered to reduce the real value of the debt,
- firms and individuals holding state bonds can be relieved of them by the taxman,

In contrast to these direct measures, austerity imposed on the non-bondholding classes, is much less effective. Since liabilities between sectors of an economy must sum to zero, taxing those too poor to save will only reduce state debt to the extent that the poor are driven to take out increased personal loans. The increase in the liabilities of the poor then compensates for the fall in state debt. The poor, however, are not very credit worthy as the sub-prime mortgage collapse showed.

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